Chapter 28 Physical Optics: Interference and Diffraction

Chapter Outline

28-1 Superposition and Interference

28-2 Young's Two-Slit Experience

28-4 Diffraction

28-5 Resolution

28-4 Diffraction

The water wave demonstrates that the wave can spread out in all direction after it goes through a small gap/slit: Huygens's Principle for light diffraction.

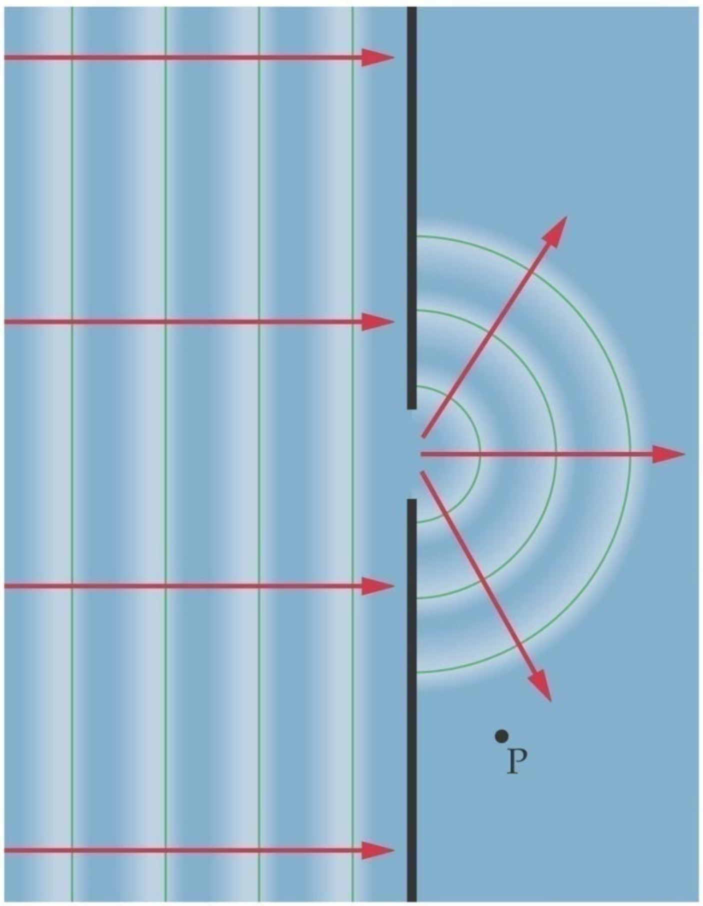

Figure 28-17
Diffraction of Water Waves

Similar, when the light goes through narrow slit, the light is spread out in all directions, and a diffraction pattern of bright - dark fringes is formed.

Diffraction Pattern on screen

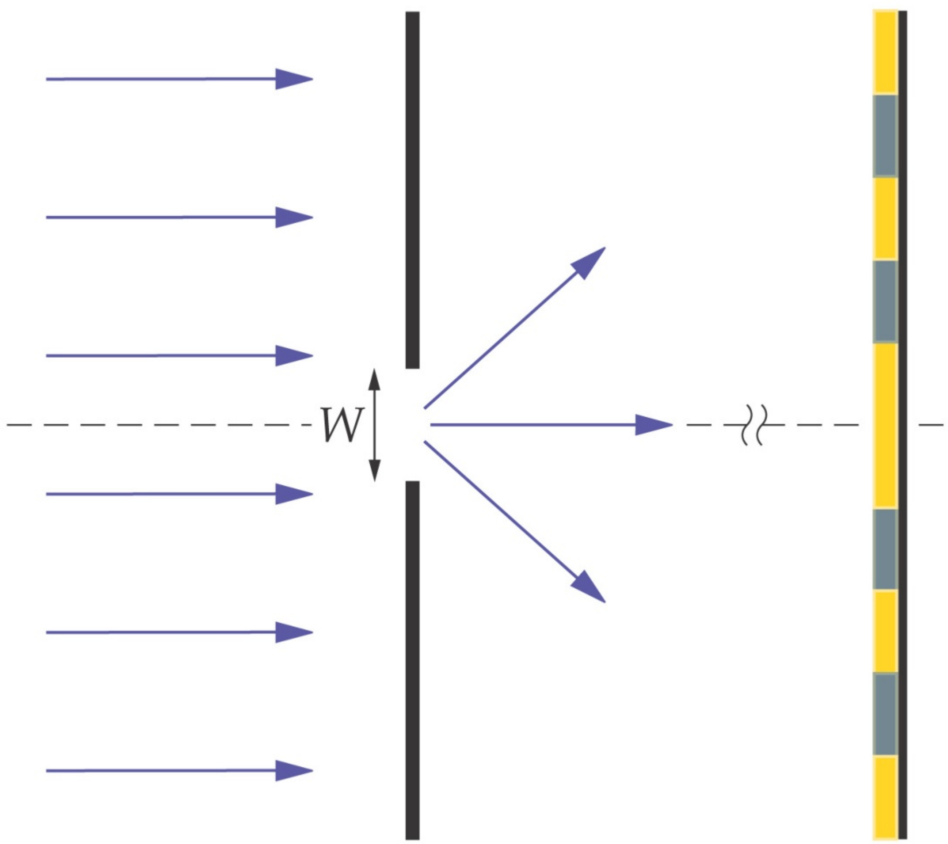

Figure 28-18
Single-Slit Diffraction

To find the first dark fringe, we divide the slit into two regions.
First dark fringe happens when:

$$\frac{w}{2}\sin\theta = \frac{\lambda}{2}, \qquad i.e. \quad w\sin\theta = \lambda$$

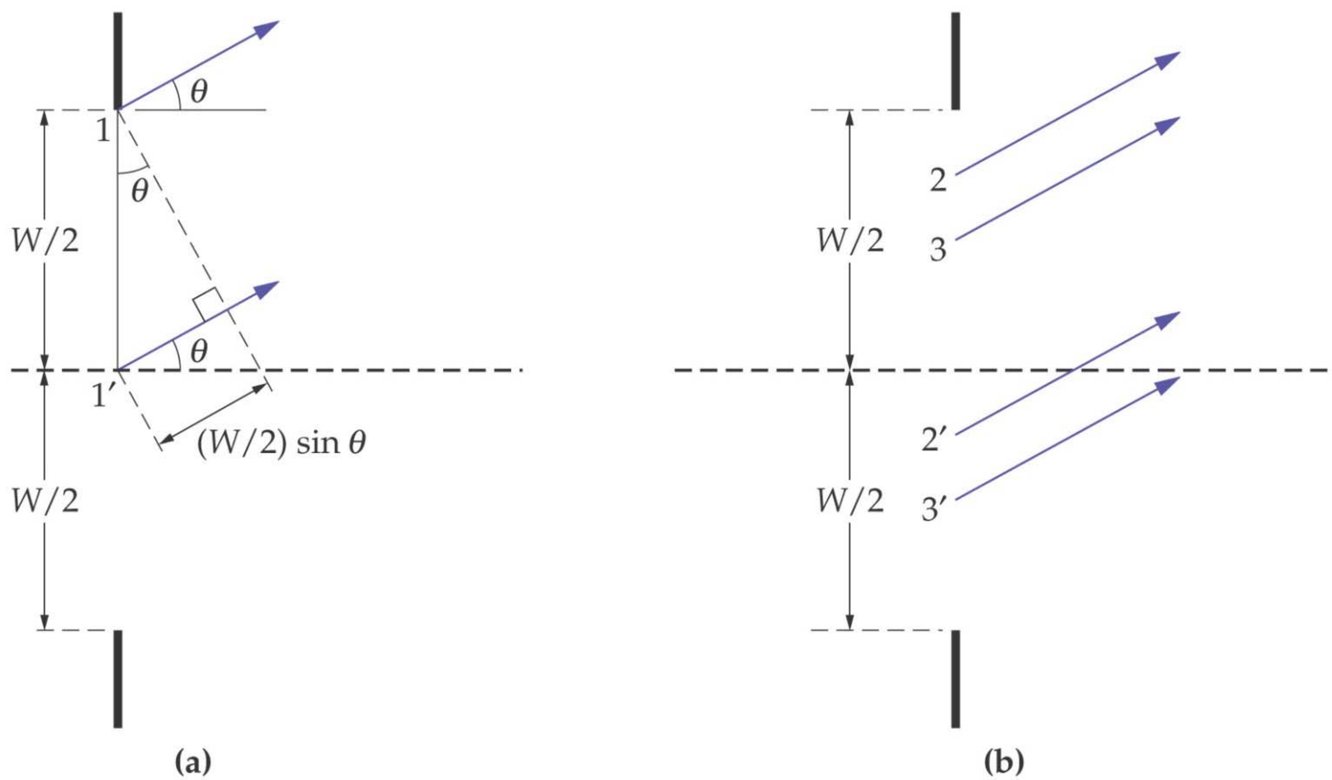

Figure 28-19

Locating the First Dark Fringe in Single-Slit Diffraction

To find the second dark fringe, we divide the slit into four regions.
The second dark fringe happens, when:

$$\frac{w}{4}\sin\theta = \frac{\lambda}{2}, \qquad i.e. \quad w\sin\theta = 2\lambda$$

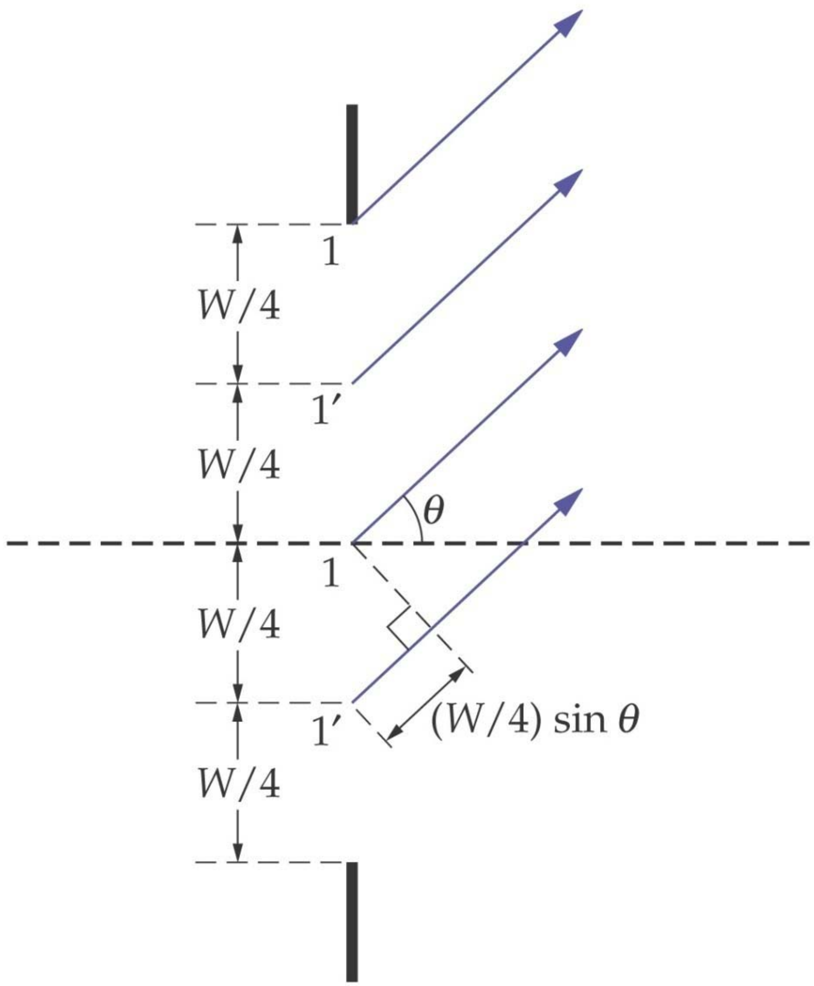

Figure 28-20
Locating the Second Dark Fringe in Single-Slit Diffraction

To find the third dark fringe, we divide the slit into six regions. The third dark fringe happens, when:

$$\frac{w}{6}\sin\theta = \frac{\lambda}{2}, \qquad i.e. \quad w\sin\theta = 3\lambda$$

And so on: divide the slit as even number regions….

Condition for Dark Fringe in Single-Slit Interference

$$W\sin\theta = m\lambda, \qquad m = \pm 1, \pm 2, \pm 3, \cdots \qquad (28-12)$$

M=1, first dark fringe;

M=2, second dark fringe;

……

The central bright fringe of diffraction slit has a angular width:

$$\theta_{Central\quad fringe} \approx 2\frac{\lambda}{W} \qquad (28-13)$$

Unit: radian

Exercise 28-2

Monochromatic light passes through a slit of width 1.2×10^{-5} mm. If the first dark fringe of the resulting diffraction pattern is at angle $\theta = 3.25^{\circ}$, what is the wavelength of the light?

Solution:

$$\text{since,}\quad W\sin\theta = m\lambda,$$

$$(1.2\times10^{-5}\,mm)\sin 3.25° = (1)\lambda$$

$$\lambda = 6.80\times10^{-7}\,mm = 680nm$$

CONCEPTUAL CHECKPOINT 28–3

If the width of the slit through which light passes is reduced, does the central bright fringe **(a)** become wider, **(b)** become narrower, or **(c)** remain the same size?

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Reasoning and Discussion

It might seem that making the slit narrower will cause the diffraction pattern to be narrower as well. Recall, however, that the diffraction pattern is produced by waves propagating from all parts of the slit. If the slit is wide, the incoming wave passes through with little deflection. If it is small, on the other hand, it acts like a point source, and light is radiated over a broad range of angles. Therefore, the smaller slit produces a wider central fringe.

This result is also confirmed by considering Equation 28–13, where we see that a smaller value of W results in a wider central fringe.

Answer:

(a) The central bright fringe is wider.

Example 28-5

Light with a wavelength of 511 nm forms a diffraction pattern after passing through a single slit of width 2.20×10^{-6}m. Find the angle associated with (a) the first and (b) the second dark fringe above the central bright fringe.

Solution:

(a) First Dark Fringe, m=1

$$\text{since} \quad W \sin\theta = m\lambda,$$

$$(2.2\times10^{-6}\,m)\sin\theta = (1)(511\times10^{-9}\,m)$$

$$\theta = 13.4^\circ$$

(b) Second Dark Fringe, m=2

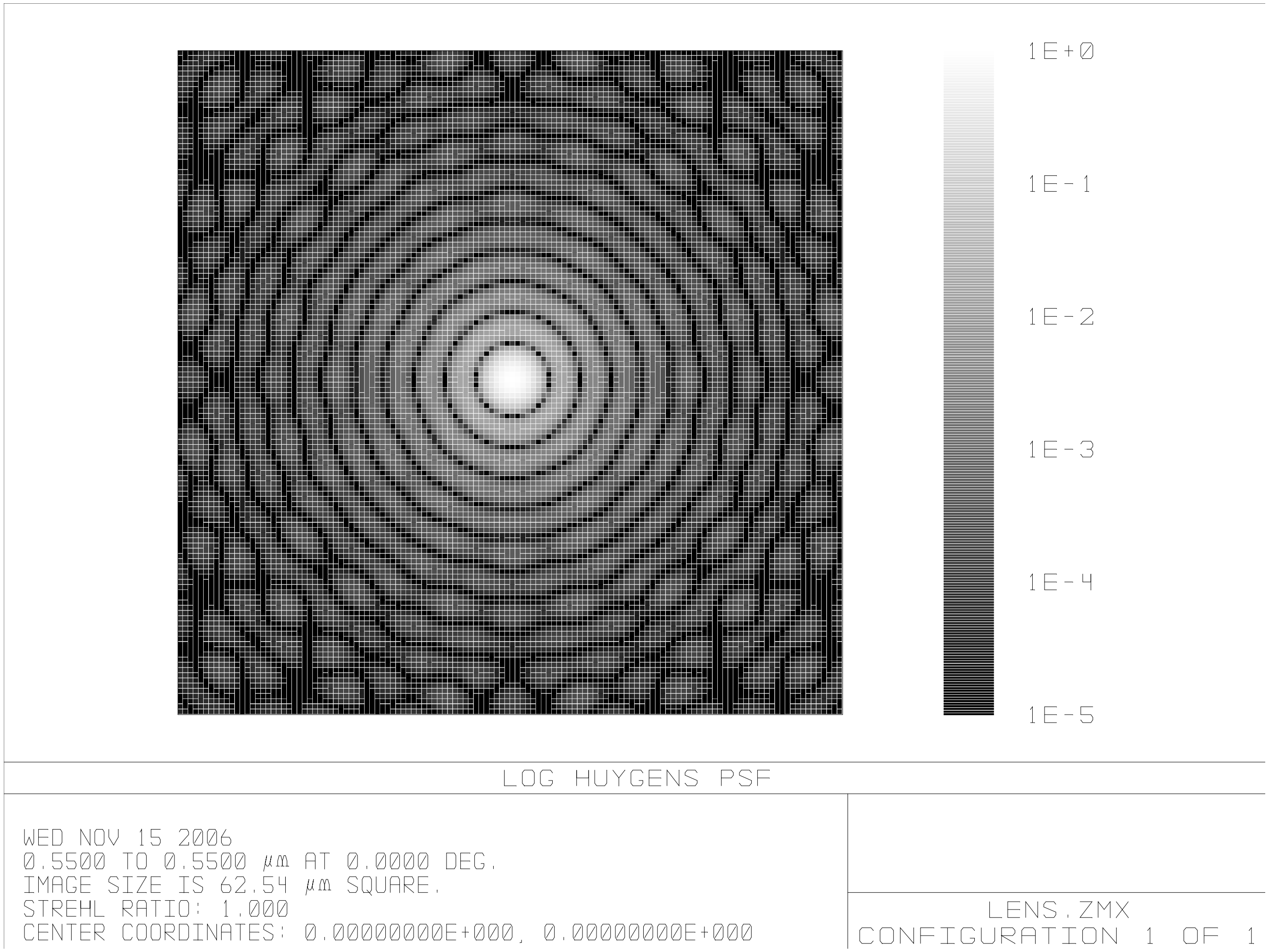
$$\text{since} \quad W \sin\theta = m\lambda,$$

$$(2.2\times10^{-6}\,m)\sin\theta = (2)(511\times10^{-9}\,m)$$

$$\theta = 27.7^\circ$$

28-5 Resolution

Diffraction pattern of a circular aperture



It can be shown:

The angle of First Dark Fringe for the Diffraction Pattern of a Circular Aperture is (from the bright point center to the first dark fringe)

$$\sin\theta = 1.22\frac{\lambda}{D} \qquad (28-14)$$

Rayleigh’s Criterion for two-point sources:

If the first dark fringe of one circular diffraction pattern passes through the center of a second diffraction pattern, the two sources will appear to be a single source, and therefore the two objects can not be seen as separate source and can not be resolved.

Two objects can be seen as separate source only if their angular separation is great than the following minimum:

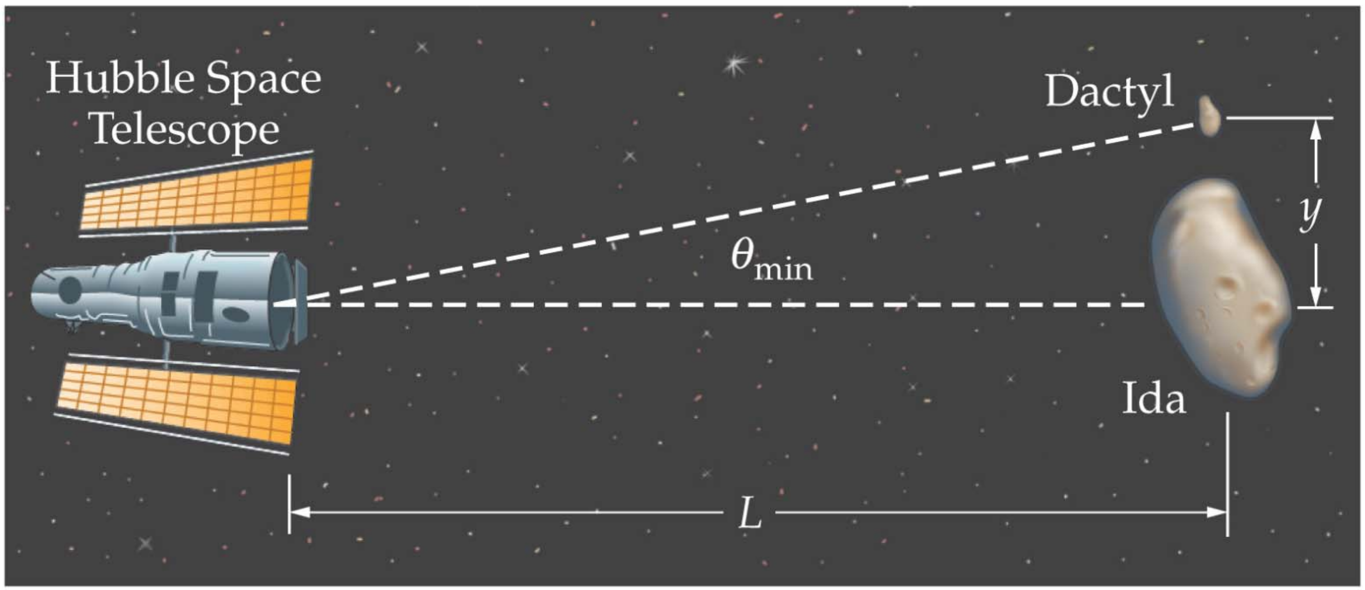
Rayleight’s Criterion (minimum angular resolution)

$$\theta_{\min} = 1.22 \frac{\lambda}{D} \qquad (28-15)$$

Unit: radian

Active Example 28-3

The asteroid Ida is orbited by its own small “moon” called Dactyl. If the separation between these two asteroids is 2.5 km, what is the maximum distance at which the Hubble Space Telescope (aperture diameter = 2.4m) can still resolve them at 550-nm wavelength?



Active Example 28-3
Resolving Ida and Dactyl

Solution

1) The minimum angular resolution is

$$\theta_{\min} = 1.22\frac{\lambda}{D} = 1.22\left(\frac{550\times10^{-9}m}{2.4m}\right) = 2.8\times10^{-7} \qquad radian$$

2) The maximum distance is

$$L = \frac{y}{\tan\theta_n} = \frac{2.5km}{\tan(2.8\times10^{-7}rad)} = 8.9\times10^{6}km$$

Summary

Condition for Dark Fringe in Single-Slit Interference

$$W \sin\theta = m\lambda, \qquad m = \pm 1, \pm 2, \pm 3, \cdots \qquad (28-12)$$

Two object can be seen as separate source only if their angular separation is great than the following minimum:

Rayleight's Criterion

$$\theta_{\min} = 1.22 \frac{\lambda}{D} \qquad (28-15)$$

Unit Radian